Exercise 16

Differentiate the function.

$$y = \ln|1 + t - t^3|$$

Solution

Take the derivative of the function.

$$\begin{aligned}
y' &= \frac{d}{dt}\left(\ln|1 + t - t^3|\right) \\
&= \frac{d}{dt}\left[\ln\sqrt{(1 + t - t^3)^2}\right] \\
&= \frac{1}{\sqrt{(1 + t - t^3)^2}} \cdot \frac{d}{dt}\sqrt{(1 + t - t^3)^2} \\
&= \frac{1}{\sqrt{(1 + t - t^3)^2}} \cdot \frac{1}{2}[(1 + t - t^3)^2]^{-1/2} \cdot \frac{d}{dt}(1 + t - t^3)^2 \\
&= \frac{1}{\sqrt{(1 + t - t^3)^2}} \cdot \frac{1}{2}[(1 + t - t^3)^2]^{-1/2} \cdot 2(1 + t - t^3) \cdot \frac{d}{dt}(1 + t - t^3) \\
&= \frac{1}{\sqrt{(1 + t - t^3)^2}} \cdot \frac{1}{2}[(1 + t - t^3)^2]^{-1/2} \cdot 2(1 + t - t^3) \cdot (1 - 3t^2) \\
&= \frac{1}{\sqrt{(1 + t - t^3)^2}} \cdot \frac{1}{\sqrt{(1 + t - t^3)^2}} \cdot (1 + t - t^3) \cdot (1 - 3t^2) \\
&= \frac{(1 + t - t^3)(1 - 3t^2)}{(1 + t - t^3)^2} \\
&= \frac{1 - 3t^2}{1 + t - t^3}
\end{aligned}$$